EXERCISE – II HINTS & SOLUTIONS

Sol.1 **B,C,D**

$|(z_1 + z_2)^2| = |z_1|^2 + |z_2|^2$

$(z_1 + z_2)\,(\bar{z}_1 + \bar{z}_2) = |z_1|^2 + |z_2|^2$

$|z_1|^2 + |z_2|^2 + z_1\,\bar{z}_2 + z_2\,\bar{z}_1 \neq |z_1|^2 + |z_2|^2$

$z_1\,\bar{z}_2 + z_2\,\bar{z}_1 = 0$

$z_1\,\bar{z}_2 = -z_2\,\bar{z}_1$

$\frac{z_1}{z_2} = -\frac{\bar{z}_1}{\bar{z}_2}$

$\frac{z_1}{z_2} + \frac{\bar{z}_1}{\bar{z}_2} = 0$ So $\frac{z_1}{z_2}$ purely imaginary

Sol.2 **A,B,C**

$|z - i| + |z + i| = k$ $k > 0$

for being ellipse $|z_1 - z_2| < 2a$

$|-2i| < k$

$k > 2$ an ellipse of $k > 2$

If $k = 5$ ellipse. If $k = 2$ then line legment

Sol.3 **A,C,D**

$|PF_1 - PF_2| = 2a$ hyperbola

$(0 < k < 2)$

$|\,|z + i| - |z - i|\,| = k$

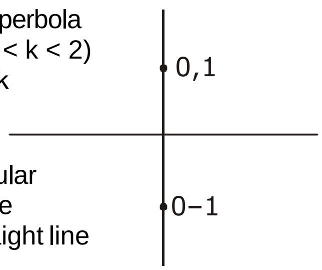
if $k = 0$

$|z + i| = |z - i|$

if line on prependicular

bisector straight line

if $k = 2$. Pair of straight line

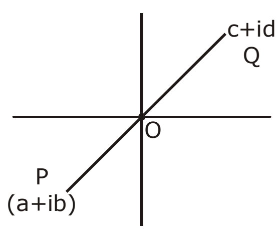


Sol.4 **A,B**

$OP = OQ$

$|a + ib| = |c + id|$

$\frac{a+c}{2} = \frac{b+d}{2}$



Sol.5 **A,B,C,D**

max (arg z) = $\pi/2$

Now $|z| = 55 - 1$

max $|z| = 55 + 1$

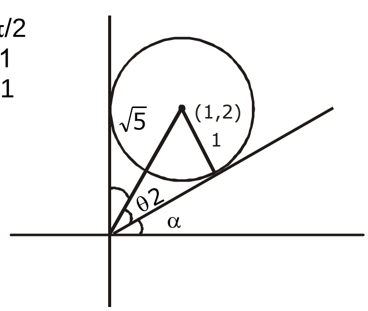
$\tan\theta = 1/2$

$\alpha = 90 - 2\theta$

$\tan\alpha = \cot 2\theta$

$= 1/\tan 2\theta$

$= 3/4$



Sol.6 **B,C**

$\frac{z^2}{|z|^2} + \frac{z}{|z|} + 1 = 0$

$t^2 + t + 1 = 0$ $\frac{z}{|z|} = t$

$t = w, w^2$ $\frac{z}{|z|} = w,\ \frac{z}{|z|} = t = w^2$

$z = |z|\,w,\ z = |z|\,w^2$

$z = kw,\ kw^2$

Sol.7 **A,B,C**

$x = e^{i\theta}$ $y = e^{i\phi}$

$x + 1/x = 2\cos\theta$ $2\cos\phi = y + 1/y$

$x^n + 1/x^n = e^{in\theta} + e^{-in\phi} = 2\cos n\theta$

$\frac{x}{y} + \frac{y}{x} = e^{i(\theta-\phi)} + e^{-i(\theta-\phi)} = 2\cos(\theta + \phi)$

Sol.8 **B,D**

$\frac{(1+i)^{2n}}{2^n} + \frac{(1-i)^{2n}}{2^n}$

$\frac{((1+i)^2)^n}{2^n} + \frac{((1-i)^2)^n}{2^n}$

$\frac{(2i)^n}{2^n} + \frac{(-2i)^n}{2^n}$

$(1)^n + (-i)^n$

$(1)^n + \left(\frac{-i}{i}\times i\right)^n$

$(1)^n + \left(\frac{1}{i}\right)^n$

$(1)^n + (i)^{-n}$

$\frac{2^n}{(2i)^n} + \frac{2^n}{(-2i)^n} = \frac{1}{i^n} + \frac{1}{(i)^{-n}} = i^{-n} + i^n$

Sol.9 **A,D**

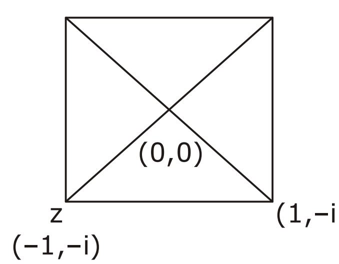
$z = -1 - i$

$iz = i\,((-1 - i)$

$= 1 - i$

$B = iz$

$D = -iz$



Sol.10 **A,C,D**

$g(x^3) + x(h(x^3)$ is divisible by $x^2 + x + 1$ so ω, $\omega 2$ is factor

$g(\omega^3) + \omega^6(h\omega^6) = 0$

$g(1) + h(1) = 0$

$g(1) = -h(1)$

$g(1) = h(1) = 0$